The Ferrite-Loaded Waveguide Discontinuity Problem

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***Abstract*—A method is presented to obtain the scattering matrix of two-port junctions consisting of a waveguide inhomogeneously loaded with ferrite. Some approximations are discussed and numerical results obtained for the dielectric-loaded twin-slab phase shifter in rectangular waveguide.**

I. INTRODUCTION

DISCONTINUITY problems in anisotropically loaded waveguides are important from both the theoretical and practical point of view (impedance matching, accurate measurement of ferrite parameters, etc.). In this paper we analyze a two-port junction with the objective of solving the field discontinuity problem and obtaining the scattering matrix. Two types of discontinuities are considered as follows. 1) Single aperture discontinuity: a waveguide is isotropically loaded for $z < 0$, and totally or partially loaded with anisotropic media for $z > 0$. The plane $z = 0$ is the aperture plane. 2) Double aperture discontinuities: a finite length of waveguide is totally or partially loaded with anisotropic media; the rest of the waveguide is homogeneously or inhomogeneously loaded with isotropic media. The two aperture planes coincide with the ends of the anisotropic section of the waveguide.

The formulation is restricted to the class of problems in which the ferrite section (the part of the waveguide loaded with ferrite) satisfies the following requirements.

1) The applied H_{dc} field is perpendicular to the direction of propagation.

2) The incident mode is a TE_{n0} mode. This is necessary to ensure that only modes of the TE_{n0} set are excited in the ferrite section, i.e., modes with only one E component (parallel to H_{dc}) and with no variation along the direction of the applied H_{dc} field.

3) The dielectric–ferrite interfaces in the waveguide are parallel to the applied H_{dc} field. This ensures that only TE_{n0} modes are excited.

Examples of the class of configurations satisfying these conditions are shown in Fig. 1. In rectangular waveguides the loading spans the narrow dimension of the guide. For all these problems, the isotropic sections can be empty waveguides (of the same cross section) or waveguides loaded with dielectric lossless slabs. The ferrite loading the guide is not assumed lossless.

Manuscript received May 2, 1974; revised August 28, 1974. This work was supported by the National Science Foundation under Grant GK-31591.

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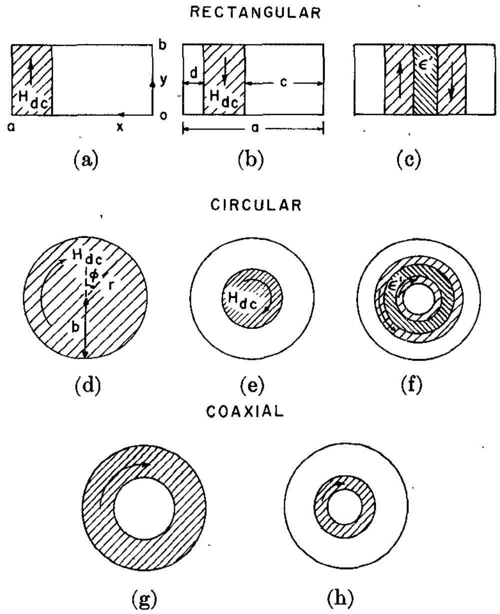

Fig. 1. Examples of configurations that can be analyzed by the method.

The literature dealing with discontinuity problems in anisotropically loaded waveguides is sparse. Apart from early work on small discontinuities and general discussions of the subject, the first attempt at a solution was made by Suhl and Walker [1]. These authors applied a perturbation method to the problem of circular waveguides filled with longitudinally magnetized ferrite. An explicit formulation of the mode-matching method was obtained by Epstein [2], who analyzed an infinite linear system of equations, the unknowns being the reflection coefficients; he then pointed out the difficulty of solving the system in an exact way. Sharpe and Heim [3] dealt with the same problem by solving an integral equation and obtaining a Neumann series expansion for the aperture electric field (and the equivalent circuit reactance). Shortly thereafter, Lewin [4] obtained a closed-form solution of the integral equation and pointed out a paradoxical result. Lewin's paradox attracted a number of researchers; their results are analyzed by Lewin, who has summarized the remaining problems posed by the paradox [5]. Bresler [6] formulated a general approach to the single-discontinuity problem, leading to an integral equation which was solved by variational methods. His method constitutes the only general systematic approach published to date. He also considered the double-discontinuity problem when the length of the anisotropic section is large enough to allow neglecting the coupling between apertures due to the beyond-cutoff modes.

Numerical and experimental results for the completely filled rectangular waveguide have been published by O'Brien [7], [8] and Gagné [9].

II. THE SINGLE-APERTURE DISCONTINUITY PROBLEM

A. Formulation of the Problem

Assuming an incident TE_{10} mode of unit amplitude, the transverse electric and magnetic fields in the isotropic section ($z \leq 0$) are

$$\text{electric field:}\quad E_1(s)\exp(-j\beta_1 z) + R_1E_1(s)\exp(j\beta_1 z) + \sum_{n=2}^{\infty} R_nE_n(s)\exp(j\beta_n z) \quad (1a)$$

$$\text{magnetic field:}\quad -Y_1E_1(s)\exp(-j\beta_1 z) + R_1Y_1E_1(s)\cdot\exp(j\beta_1 z) + \sum_{n=2}^{\infty} R_nY_nE_n(s)\exp(j\beta_n z) \quad (1b)$$

where s is the transverse coordinate, the Y_n are the mode admittances, and R_n are the unknown reflection coefficients. The fields in the ferrite section ($z \geq 0$) are

$$\text{electric field:}\quad \sum_{n=1}^{\infty} T_nE_n^+(s)\exp(-j\beta_n^+ z) \quad (2a)$$

$$\text{magnetic field:}\quad \sum_{n=1}^{\infty} T_nH_n^+(s)\exp(-j\beta_n^+ z) \quad (2b)$$

where T_n are the unknown transmission coefficients.

Assuming that these TE_{n0} sets are complete, the boundary conditions at the aperture plane $z = 0$ (continuity of transverse E and H) become

$$E_1(s) + \sum_{n=1}^{\infty} R_nE_n(s) = \sum_{n=1}^{\infty} T_nE_n^+(s) \equiv \epsilon(s) \quad (3a)$$

$$-Y_1E_1(s) + \sum_{n=1}^{\infty} R_nY_nE_n(s) = \sum_{n=1}^{\infty} T_nH_n^+(s). \quad (3b)$$

Equation (3) is used to define the aperture electric field $\epsilon(s)$, which is the unknown transverse electric field at $z = 0$.

Equation (3) is formally the same as the one for isotropic discontinuity problems: in both cases the objective is to solve for R_n, T_n [or to find $\epsilon(s)$]. However, we cannot follow the method used in isotropic problems because: 1) the E_n^+ set is not orthogonal, 2) the H_n^+ set is not orthogonal, and 3) H_n^+ is not simply related to E_n^+ by means of an admittance. Although the $E_n^+(s)$ set is not orthogonal, a valid "biorthogonality" relationship exists, involving both $E_n^+(s)$ and $H_n^+(s)$ [10]–[12]; this relationship was used by Bresler [6] to obtain an integral equation [involving both $\epsilon(s)$ and the magnetic aperture field] in a properly defined product space, when the ferrite losses are zero.

We follow a different approach leading to an integral equation which involves only $\epsilon(s)$. The main obstacle is the nonorthogonality of the set $E_n^+(s)$. This can be circumvented in several ways; for example, since the $E_n^+(s)$ set is linearly independent, it can be orthogonalized by the Gram–Schmidt method. While this would be a perfectly valid approach, it has the disadvantage of introducing a new set of orthogonal functions which do not correspond in general to any identifiable physical mode. Furthermore, it often happens that the beyond-cutoff (i.e., higher order) modes of the $E_n^+(s)$ set are not known and their determination is, in general, far from trivial since their propagation factors are generally complex [10]. It was then decided to try first to expand the E_n^+ fields in terms of an orthogonal set of modes, called the equivalent dielectric guide (EDG) set which are the modes supported by a waveguide in which the ferrite has been replaced by a dielectric of the same permittivity. The main advantage of this expansion is that it leads naturally to the "dielectric approximation" in which the higher order modes in the ferrite section are replaced by the higher order modes in the equivalent dielectric waveguide. Consequently, we set

$$E_n^+(s) = \sum_{i=1}^{\infty} \alpha_{in}'E_{di}(s) \quad (4)$$

where the subindex "d" denotes the modes of the EDG guide. Since the set $E_{di}(s)$ is orthogonal, we have

$$\alpha_{in}' = (E_n^+,E_{di})/(E_{di},E_{di}). \quad (5)$$

The scalar products appearing in (4) and (5) are given by the integral

$$(f,g) = \int_s f(s)g^*(s)\,ds \quad (6)$$

where the integral is a surface integral over the cross section of the waveguide, and the symbol * indicates the complex conjugate.

The electric field in the ferrite region can now be written in terms of α_{in}' by substituting (3a) as follows:

$$\sum_{n=1}^{\infty} T_nE_n^+(s) = \sum_{i=1}^{\infty} \alpha_iE_{di}(s) \quad (7)$$

where

$$\alpha_i = \sum_{n=1}^{\infty} \alpha_{in}'T_n, \qquad i = 1\cdots\infty. \quad (8)$$

Since the $E_n^+(s)$ set is linearly independent, the matrix (α') has an inverse. Furthermore, if this matrix is truncated at an arbitrary value of i, the resultant square matrix will also have an inverse. We can then write

$$T_n = \sum_{i=1}^{\infty} \psi_{ni}\alpha_i \quad (9)$$

where either $i = 1\cdots$ in the exact case, or $i = 1\cdots M$ in the approximate case where the expansion matrix (α') has been truncated. Substituting (7) into (3a),

$$E_1(s) + \sum_{1}^{\infty} R_nE_n(s) = \sum_{1}^{\infty} \alpha_iE_{di}(s) \equiv \epsilon(s). \quad (10)$$

From now on, we can follow the same procedure used for isotropic problems to obtain the following integral equation:

$$E_1(s) = \int_s G(s,s')\epsilon(s')\,ds' \qquad (11)$$

where

$$G(s,s') = \sum_{n=1}^{\infty} \frac{Y_n}{2Y_1(E_n,E_n)} E_n(s)E_n^*(s')$$

$$- \sum_{n,i=1}^{\infty} \frac{\psi_{ni}}{2Y_1(E_{di},E_{di})} H_n^+(s)E_{di}^*(s'). \qquad (12)$$

B. Solution of the Integral Equation

It is evident that an exact closed-form solution of the integral equation (11) cannot be expected, except perhaps for simple configurations such as the completely filled rectangular guide of Fig. 1(a). We are, therefore, led to consider approximate methods. Of these, the most attractive seems to be a variational solution, although we only have at our disposal stationary (rather than extremun) principles, due to the non-Hermitian nature of the kernel (12). A stationary expression for R is given by the Schwinger–Levine stationary principle [14]:

$$I + R_1 = \frac{(\epsilon,E_1)(E_1,\epsilon_a)}{(E_1,E_1)(G\epsilon,\epsilon_a)} \qquad (13)$$

where the original integral equation (11) is written $\mathcal{G}\epsilon = E_1$, and ϵ_a is the solution of the adjoint equation $\mathcal{G}^*\epsilon_a = E_1$. As is generally the case with variational methods, it is difficult to know in advance which trial functions will give best results, and in this case the problem is compounded by the need for estimating both ϵ and ϵ_a. To generate trial functions we have adopted the generalized Ritz–Rayleigh method, in which both trial functions are expressed as linear combinations of known arbitrary functions with unknown coefficients. In principle, we could use as arbitrary functions EDG modes, isotropic modes, ferrite modes, or combination of these. We have decided to expand ϵ and ϵ_a in terms of the isotropic set E_i:

$$\epsilon(s) = \sum_{i=1}^{N} R_i' E_i(s) \qquad (14a)$$

$$\epsilon_a(s) = \sum_{i=1}^{N} b_i E_i(s) \qquad (14b)$$

where the coefficients R_i', b_i are unknown. If we substitute (14) into (13) and then equate to zero the derivative of the resulting expression with respect to the b_i, we obtain a set of linear equations for the unknown coefficients R_i':

$$\sum_{i=1}^{N} R_i'(\mathcal{G}E_1,E_h) = (f,E_h), \qquad h = 1,2,\cdots N. \qquad (15)$$

As a consequence of our choice of expansion functions, the same system is obtained by straight algebraization of the integral equation, i.e., by substituting (14a) directly into (11), since $R_1' = 1 + R_1$ and $R_n' = R_n$ for $n \geq 2$. The advantage of our choice is that upon solution of the system (15) we obtain all the R_n instead of only R_1. We can rewrite this system as follows:

$$\sum_{i=1}^{N} \gamma_{ih} R_i' = y_1 \delta_{ih} \qquad (16)$$

where

$$\gamma_{ih} \equiv \frac{Y_i}{2Y_1} y_i \delta_{ih} - \sum_{n=1}^{N} \sum_{j=1}^{M} \frac{c_{nh}\psi_{nj}(E_i,E_{dj})}{2Y_1(E_{dj},E_{dj})} \qquad (17a)$$

$$y_i \equiv (E_i,E_i) \qquad (17b)$$

$$c_{nh} \equiv (H_n^+,E_h) \qquad (17c)$$

$$\delta_{ij} = \text{Kronecker's symbol.}$$

The problem is now reduced to a linear system of equations, and we turn our attention to methods of estimating the accuracy of the solution.

C. Accuracy of the Solution—the Scattering Matrix of the Junction

Heller [16] has shown that the scattering matrix of a lossless two-port junction has the form

$$S = \begin{pmatrix} a \exp(j\alpha) & (1-a^2)^{1/2} \exp[j(\alpha - \phi)] \\ -(1-a^2)^{1/2} \exp[j(\delta + \phi)] & a \exp(j\delta) \end{pmatrix} \qquad (18)$$

where a, α, ϕ, and δ are real numbers.

The scattering matrix for the single aperture problem is

$$S = \begin{pmatrix} R_1 & T_2(P_1/P_{1f})^{1/2} \\ T_1(P_{1f}/P_1)^{1/2} & R_2 \end{pmatrix} \qquad (19)$$

where R_2, T_2 are the reflection and transmission coefficients when a unit amplitude mode is incident from the ferrite section. These can be obtained in an entirely analogous manner to R_1, T_1 and thus their derivation is not repeated. P_1 and P_{1f} are the power carried by the unit amplitude TE_{10} mode in the isotropic and ferrite sections, respectively. Consequently, (19) has to be of the form given by (18); in particular when the junction is lossless we must have conservation of energy, i.e.,

$$T_1(P_{1f}/P_1)^{1/2} = (1 - R_1^2)^{1/2}. \qquad (20)$$

Another check on the solution is to compute the aperture electric field from both sides of the aperture plane and to see if the real and imaginary part of $\epsilon(s)$ match over the whole plane. This is the only ready check available when the junction is lossy.

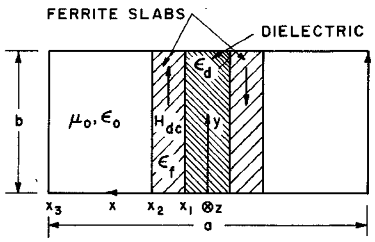

Fig. 2. Theoretical model for the remanence twin-slab phase shifter in rectangular waveguide.

III. AN EXAMPLE: NUMERICAL RESULTS FOR TWIN-SLAB PHASE SHIFTER

The method developed in the previous sections has been applied to the junction between a perfectly conducting empty rectangular waveguide and a rectangular waveguide of the same dimension loaded from $z = 0$ to $z = \infty$ with two symmetrically placed slabs of lossless ferrite magnetized a remanence in opposite directions and with dielectric loading in between (see Fig. 2). This configuration is the widely used theoretical model of the dielectric loaded twin-slab remanence latching phase shifter. The normalized dimensions have been taken from [17]

$$x_1/\lambda_0 = 0.02, \quad x_2/\lambda_0 = 0.07, \quad x_3/\lambda_0 = 0.3805,$$
$$\epsilon_d/\epsilon_0 = 13, \quad \epsilon_f/\epsilon_0 = 12.$$

The linear system (16) was solved for this configuration with $N = 20$. The results appear in Figs. 3 and 4. The broken lines in both correspond to the dielectric approximation, in which the ferrite higher order modes are replaced by the EDG higher order modes. Fig. 3 shows the absolute value of the reflection coefficient as a function of normalized remanent magnetization; also shown is the energy residual, expressed as a percentage of the incident power. As the remanent magnetization is increased, the reflection coefficient decreases, contrary to what could be expected intuitively since the fields in the ferrite depart more from the dielectric limit. In view of the relatively large error resulting from the dielectric approximation, it was decided to improve it by retaining the first two modes of the ferrite section, the rest being replaced by the dielectric modes as before. The results appear as solid lines in Figs. 3 and 4. There is a marked reduction in error, but the results suggest that the validity of the dielectric approximation is quite limited. The complex propagation factor for the first higher order mode was obtained by a method originally used by Gardiol [18] in a similar problem. Fig. 4(a) shows the incident mode, transmitted mode, and the aperture field in the dielectric limit. Due to the symmetry of the fields, only one-half of the waveguide is shown. Fig. 4(b) and (c) shows the transmitted mode and the absolute value of the aperture field when the normalized magnetization equals ± 0.5, respectively. The departure from the TE_{10} transmitted fields indicates the substantial contribution of higher order modes excited at the junction. These plots were obtained by summing 20 modes on both sides of the junction. Their real and imaginary parts agree within 1 percent of each other over the aperture plane.

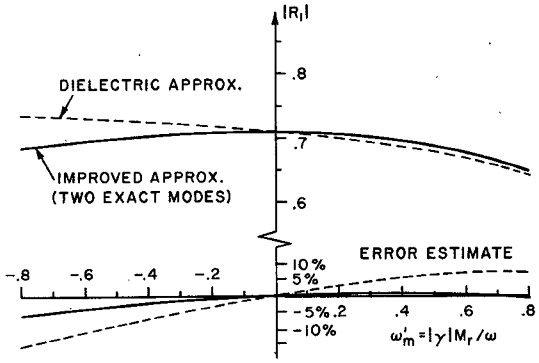

Fig. 3. Single aperture discontinuity. Absolute value of reflection coefficient and energy residual as a function of normalized remanent magnetization (clockwise or anticlockwise).

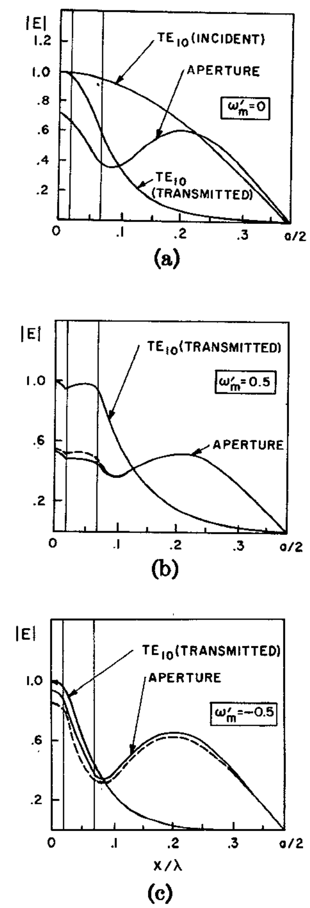

Fig. 4. (a)–(c) Electric field versus distance inside the waveguide. (a) shows the incident and transmitted TE_{10} modes and the aperture field in the dielectric limit; (b)–(c) shows the transmitted field and the absolute value of the aperture field for clockwise and anticlockwise magnetization. Broken lines correspond to the dielectric approximation. Because of symmetry, only half the waveguide is shown.

IV. THE DOUBLE-APERTURE DISCONTINUITY PROBLEM

We now assume that the anisotropic section has a finite length $L = 2l$ (Fig. 5). The mode expansions in the three regions can be written as follows.

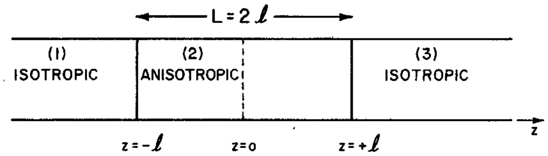

Fig. 5. Double aperture discontinuity. Definition of parameters.

Region (1): $z < -l$: Same as 1a–b.

Region (2): $-l < z < l$:

$$\text{electric:} \quad \sum_{n=1}^{\infty} A_n \exp(-j\beta_n^+ l) E_n^+(s) \exp(-j\beta_n^+ z) + B_n \exp(j\beta_n^- l) E_n^-(s) \exp(-j\beta_n^- z)$$

$$\text{magnetic:} \quad \sum_{n=1}^{\infty} A_n \exp(-j\beta_n^+ l) H_n^+(s) \exp(-j\beta_n^+ z) + B_n \exp(j\beta_n^- l) H_n^-(s) \exp(-j\beta_n^- z). \quad (21)$$

Region (3): $z > l$:

$$\text{electric:} \quad \sum_{n=1}^{\infty} T_n E_n(s) \exp(-j\beta_n z)$$

$$\text{magnetic:} \quad \sum_{n=1}^{\infty} -T_n Y_n E_n(s) \exp(-j\beta_n z). \quad (22)$$

The weighting factors multiplying the coefficients A_n, B_n in (21) are chosen for reasons which will be apparent later on.

Assuming that the modal expansion (21) is complete, we can proceed to satisfy the boundary conditions at both aperture planes (mode matching); with the new notation

$$T_n' = T_n \exp(-j\beta_n l), \quad \text{all } n$$

$$R_n' = R_n \exp(-j\beta_n l), \quad n \geq 2$$

$$R_1' = \exp(j\beta_1 l) + R_1 \exp(-j\beta_1 l) \quad (23)$$

these are, from the first aperture ($z = -l$)

$$\sum_{n=1}^{\infty} R_n' E_n = \sum_{n=1}^{\infty} A_n E_n^+ + B_n E_n^- \exp(2j\beta_n^- l) \quad (24a)$$

$$-2Y_1 E_1 \exp(j\beta_1 l) + \sum_{n=1}^{\infty} R_n' Y_n E_n = \sum_{n=1}^{\infty} A_n H_n^+ + B_n H_n^- \exp(2j\beta_n^- l) \quad (24b)$$

and from the second aperture ($z = l$)

$$\sum_{n=1}^{\infty} T_n' E_n = \sum_{n=1}^{\infty} A_n E_n^+ \exp(-2j\beta_n^+ l) + B_n E_n^- \quad (24c)$$

$$-\sum_{n=1}^{\infty} T_n' Y_n E_n = \sum_{n=1}^{\infty} A_n H_n^+ \exp(-2j\beta_n^+ l) + B_n H_n^-. \quad (24d)$$

From (24) we can obtain a system of coupled integral equations involving the electric aperture fields at both apertures. Their complexity precludes a closed form solution and it is better to obtain directly a linear system of equations. The question arises as to what set of orthogonal modes should be used in this process. The main advantage of the expansion in terms of the EDG fields is the simplification resulting from the dielectric approximation; in view of its limited accuracy, we opt for expanding the $E_n^\pm$ sets in terms of the isotropic E_n set. This improves the accuracy of the solution at the expense of numerical inversion of the expansion matrix.

Define

$$a_{ni} \equiv (E_n^+, E_i)$$

$$b_{ni} \equiv (E_n^-, E_i)$$

$$d_{ni} \equiv (H_n^-, E_i). \quad (25)$$

Taking the scalar product of (24a) and (24c) by E_n we obtain, respectively,

$$R_i' = \sum_{n=1}^{\infty} \frac{a_{ni}}{y_i} A_n + \frac{b_{ni}}{y_i} B_n \exp(2j\beta_n^- l) \quad (26a)$$

$$T_i' = \sum_{n=1}^{\infty} \frac{a_{ni}}{y_i} A_n \exp(-2j\beta_n^+ l) + \frac{b_{ni}}{y_i} B_n. \quad (26b)$$

This system can be inverted to yield

$$A_n = \sum_i p_{ni} R_i' + a_{ni} T_i' \quad (27a)$$

$$B_n = \sum_i r_{ni} R_i' + s_{ni} T_i'. \quad (27b)$$

Let us remark that in (26) we can neglect those exponentials that are small enough (e.g., less than 10^{-20}). This amounts of course to neglecting at apertures 1 and 2 those higher order modes (excited at apertures 2 and 1, respectively) which have negligible amplitude. This is the reason for the weighting factors in (21) and for selecting the origin $z = 0$ at the middle of the anisotropic section. Substituting (27) into (24b) and (24d) and taking the scalar product of both sides of the resulting equations by E_m, we get the following linear system of equations for R_1', T_1':

$$-2Y_1 y_1 \delta_{1m} \exp(j\beta_1 l) = \sum_{i=1}^{\infty} \{[\sum_{n=1}^{\infty} p_{ni} c_{nm} + r_{ni} d_{nm} \exp(2j\beta_n^- l)] - Y_m y_m \delta_{mi}]R_i' + \sum_{i=1}^{\infty} [\sum_{n=1}^{\infty} a_{ni} c_{nm} + s_{ni} d_{nm} \exp(2j\beta_n^- l)]T_i' \quad (28a)$$

$$0 = \sum_{i=1}^{\infty} [\sum_{n=1}^{\infty} p_{ni} c_{nm} \exp(-2j\beta_n^+ l) + r_{ni} d_{nm}]R_i' + \sum_{i=1}^{\infty} \{[\sum_{n=1}^{\infty} a_{ni} c_{nm} \exp(-2j\beta_n^+ l) + s_{ni} d_{nm}] + Y_m y_m \delta_{mi}\} T_i'. \quad (28b)$$

Thus the problem has been reduced to the linear system

(28). Note that we can again neglect in (28) the exponential factors corresponding to the higher order modes.

V. NUMERICAL RESULTS FOR TWIN-SLAB PHASE SHIFTER

The system (28) was generated and solved for the geometry previously described. A Fortran-IV program was written with the number of modes used in the expansion equal to 20, as before. The complex propagation factors of the modes in the ferrite section were obtained using the same method mentioned earlier. Typically, the propagation factors of the higher order modes would take approximately 15 s of computer time (IBM 360) for each. Solution of (28) would then take approximately 60 s for each value of L/λ_0. Throughout the computations, ω_m' was fixed at 0.5. It must be noted that only the complex propagation factors of the forward ("+") modes are needed because the ones for the reverse ("−") modes are the complex conjugates of the former [11]. In this particular example the scattering matrix can be obtained very simply by repeating the calculations with the magnetization switched, as this is equivalent to having a mode incident from the right with the magnetization unchanged. If R_{1cc} and T_{1cc} are the reflection and transmission coefficients when the magnetization is switched, the scattering matrix becomes

$$\begin{pmatrix} R_1 & T_{1cc} \\ T_1 & R_{1cc} \end{pmatrix}. \tag{29}$$

This matrix was computed for several values of L/λ_0, and is indeed of the form (18) with $\alpha = \delta$. Plots of a, α, and ϕ are shown in Fig. 6 as a function of L/λ_0. The other scales show the differential phase shift (proportional to the length) and the magnitudes L/λ^+, L/λ^-, L/λ_{av}, where λ^+ and λ^- are, respectively, $2\pi/\lambda_1^+$, $2\pi/\lambda_1^-$, and λ_{av} is $(\lambda^+ + \lambda^-)/2$. Fig. 6 shows that the minima in the absolute value of the reflection coefficient are very sharp. The location can be predicted fairly accurately because they fall near the values of $L/\lambda_{av} = n/2$, with $n = 1,2,\cdots$. Evidently, a given value of the differential phase shift might be coupled with a large value of reflection coefficient in the absence of matching structures.

Fig. 7 shows a plot of the normalized input impedance Z_i/Z_1 as a function of the normalized length L/λ_0, where Z_i is the actual impedance seen at $z = -L/2$ looking in the $+z$ direction, i.e.,

$$\frac{Z_i}{Z_1} = \frac{1 + R_1 \exp(-j\beta_1 L)}{1 - R_1 \exp(-j\beta_1 L)} \tag{30}$$

and $Z_1 = 1/Y_1$ (TE_{10} impedance). For $L = 0$ the graph starts at the center (perfect match). As L/λ_0 increases in intervals of 0.02 (numbered black dots) the graph describes a loop and reapproaches the center when $L \simeq \lambda_{av}/2$. After three loops, the influence at one aperture of the higher order modes excited at the other has practically disappeared, and the graph repeats itself. The hollow dots in Fig. 7 correspond to the dielectric limit, when $\omega_m' = 0$. Note that they fall near the corresponding black dots, because with $|\omega_m'| = 0.5$ the ferrite fields do not depart drastically from the fields that obtain in the dielectric limit. Fig. 7, therefore, furnishes the design parameter needed for an impedance matching network. Fig. 8(a)–(d) shows plots of the absolute value of the aperture fields;

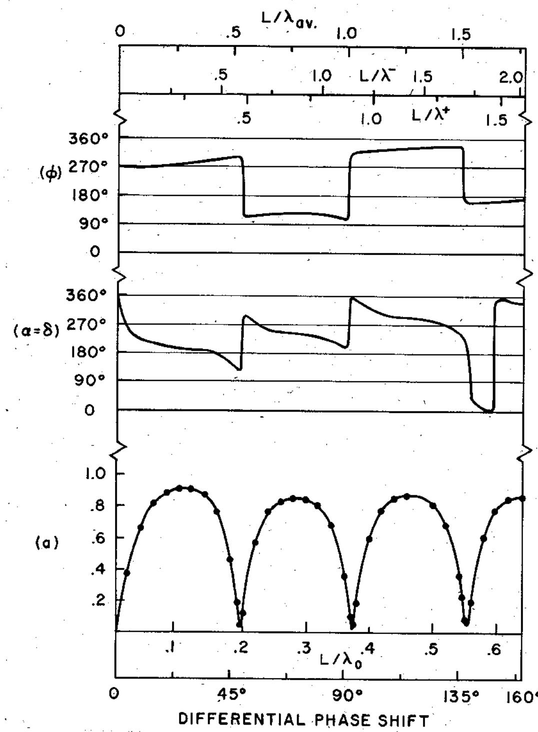

Fig. 6. Elements of the scattering matrix as a function of normalized length of ferrite section.

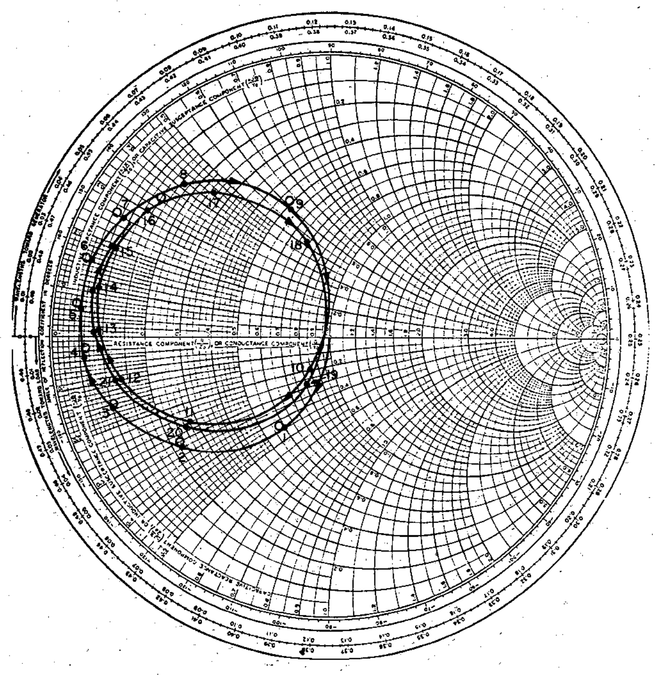

Fig. 7. Input impedance at ferrite–dielectric interface, looking in the $+z$ direction, as a function of normalized length of ferrite section. Numbered black dots correspond to those of Fig. 6. Circles correspond to dielectric limit.

Fig. 8(a) and (b) corresponds to a value of L such that the absolute value of the reflection coefficient is small; note that the fields at the first aperture resemble the transmitted TE_{10} mode. Fig. 8(c)–(d) corresponds to a large value of the reflection coefficient; the field at the first aperture is closer to zero. It should not be inferred from these plots that the actual aperture fields are nearly equal for the two magnetization states, because their real and imaginary parts differ markedly. The fields in these plots were obtained by adding up modes on both sides of the apertures, and the values coincide within 1 percent. The energy residual is also less than 0.1 percent (for any L), a result of the fact that no approximations were made to obtain the propagation factors of the higher order modes in the anisotropic section.

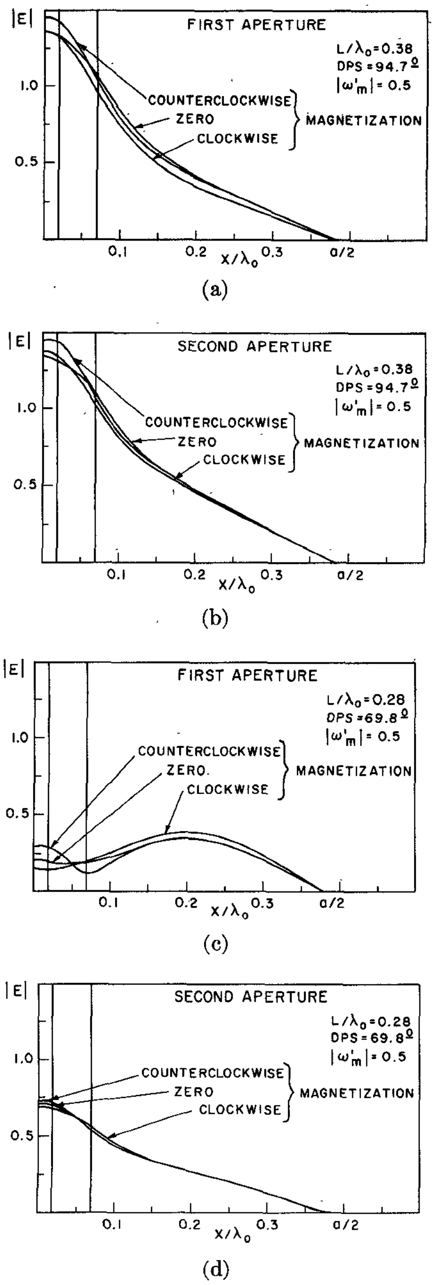

Fig. 8. (a) and (b) Absolute value of the aperture electric field as a function of normalized distance inside the waveguide. (c) and (d) Absolute value of aperture electric field for a different value of normalized remanent magnetization.

VI. CONCLUSIONS

Waveguide discontinuity problems involving finite or infinite sections of transversely magnetized ferrites are solved by a mode-matching procedure which leads to a linear system of equations. This system can be solved with good accuracy by truncation, at a reasonable matrix size. The method can be used to obtain the scattering matrix of the junction, and to obtain the design parameters for an impedance matching network. The dielectric approximation, in conjunction with expansion of the ferrite fields in terms of the EDG fields, can be used, whereupon computing time is minimized, at the expense of accuracy.

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